ON THE THEORETICAL ANALYSIS OF VIBRATORY FLAME PROPAGATION

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The condition is established under which the mutual effect between wave generation at a flame surface and periodic accelerations in the surrounding medium may become an essential feedback mechanism in the vibratory propagation of the flame along ducts.

According to [1], the mutual effect between wave generation at a flame surface and periodic accelerations acting on that flame is a second-order effect in the case of small acoustic vibrations and, therefore, the feedback-mechanism theory developed in [2, 3] cannot explain the presence of a vibratory combustion mode in ducts and for a fully developed vibratory combustion it becomes meaningless. It can be shown that this theory applies only to a specific stage in the vibratory propagation of a flame.

An analysis of this problem [1] leads to differential equations for the wave generation at a flame surface and for the acoustic vibrations:

$$\left.\begin{aligned} \frac{\partial \delta v_{sx}}{\partial t} &= -\frac{1}{\rho_s}\,\frac{\partial \delta p_s}{\partial x} \\ \frac{\partial \delta v_{sy}}{\partial t} &= -\frac{1}{\rho_s}\,\frac{\partial \delta p_s}{\partial y} \\ \frac{\partial \delta v_{sx}}{\partial x} &+ \frac{\partial \delta v_{sy}}{\partial y} = 0 \end{aligned}\right\}, \tag{1}$$

$$\left.\begin{aligned} \frac{\partial v'_s}{\partial t} &= -\frac{1}{\rho_s}\,\frac{\partial p'_s}{\partial x} \\ \frac{\partial p'_s}{\partial t} &+ \rho_s a_s^2\,\frac{\partial v'_s}{\partial x} = 0 \end{aligned}\right\} \tag{2}$$

with the following constraints at the flame surface; for the first system of equations (1)

$$\delta v_{1x} = \delta v_{2x} = \frac{\partial \delta x}{\partial t},\quad \delta x = f(t)\exp iky, \tag{3}$$

$$\delta p_1 - \delta p_2 = (\rho_1 - \rho_2)\, b\delta x \tag{4}$$

and for the second system of equations (2)

$$v'_1 = v'_2,\ p'_1 = p'_2. \tag{5}$$

As was shown in [1], conditions (3), (4), (5) are supplemented by two more:

$$h_0 \ll \frac{2\pi a_s}{\omega},\quad (\delta x)_0 \ll \frac{2\pi a_s}{\omega}, \tag{6}$$

the latter ones signifying that the amplitudes of acoustic displacement and of wave generation at the flame surface are much smaller than the wavelength of acoustic vibrations.

Two circumstances are significant here:

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a) the two systems of equations (1) and (2) are independent, the interrelation between wave generation at the flame surface and acoustic vibrations being defined by condition (4);

b) as a consequence of linearization, in deriving equations (1) and (2) second-order terms have been disregarded including those with the velocity perturbations squared in the first set and those with the acoustic velocity squared in the second set, i.e., the assumption is made that

$$\delta v_{sx} \frac{\partial \delta v_{sx}}{\partial x} \ll \frac{\partial \delta v_{sx}}{\partial t}, \; \delta v_{sx} \frac{\partial \delta v_{sy}}{\partial x} \ll \frac{\partial \delta v_{sy}}{\partial t} \text{ etc.}, \tag{7}$$

$$v_s' \frac{\partial v_s'}{\partial x} \ll \frac{\partial v_s'}{\partial t}. \tag{8}$$

The last assumption is equivalent to $v_s' \ll a_s$ [4], which coincides with the first of conditions (6).

Further analysis of the problem reduces to solving the system of equations (1) for the boundary conditions (3) and (4). On the right-hand side of (4) the product of the acoustic acceleration $b = h_0\omega^2 \cos \omega t$ and the displacement δx of the flame front is a quantity of the same order of magnitude as $v_s' \delta v_{sx}$. In the case of small acoustic vibrations, when v_s' and δv_{sx} are of the same order of magnitude, the right-hand side of (4) must, in accordance with (7), be disregarded while the boundary condition for pressure must be stated as $\delta p_1 = \delta p_2$ and the problem of analyzing the mutual effect between wave generation and acoustic vibrations becomes meaningless, i.e., we arrive at the conclusion stated in [1]. If $\delta v_{sx} \ll v_s'$, then with the fulfillment of condition $v_s' \ll a_s$ the said problem is solved entirely according to [2, 3]. Indeed, the condition of small acoustic velocity makes it permissible to analyze the acoustics (2) on the basis of a linear approximation and condition $\delta v_{sx} \ll v_s'$ suggests, as indicated by (4) and (7), that in solving Eqs. (1) the constraint on pressure should be considered in the form of (4).

The mutual effect between wave generation at the flame surface and periodic accelerations acting on that flame may, in this way, play the role of an essential feedback mechanism at the time when the vibratory flame propagation has already developed and the condition under which the theory of this mechanism according to [2, 3] is applicable becomes

$$\delta v_{sx} \ll v_s' \ll a_s. \tag{9}$$

Experiments with vibratory propagation of CO–air flames through semiopen ducts have confirmed this point of view in the following manner.

1. One observes two successive stages of vibratory flame propagation, the first related to vibrations of the unperturbed flame front and the second related to periodic variations on the entire flame surface.

2. A transition to the second stage occurs at acoustic velocities on the order of 1-3 m/sec, which satisfies condition (9).

3. From measurements of the vibration amplitude h_0 and the wavelength at the flame surface one finds the condition for wave generation to be $3.4 < 16(\rho_1-\rho_2)/(\rho_1+\rho_2)(h_0/\lambda) < 8.4$, which is in close agreement with the theoretical one [2, 3].

$$1.4 < 16\, \frac{\rho_1-\rho_2}{\rho_1+\rho_2}\, \frac{h_0}{\lambda} < 9.$$

It is noteworthy that the lower limit based on test data is 2-2.5 times higher than the theoretical one, but a discussion of this difference and what causes it is beyond the scope of this article.

NOTATION

a_s is the velocity of sound (subscript s = 1, 2 refers to the fuel mixture and to the combustion products respectively);
b is the acoustic acceleration;
h_0 is the amplitude of acoustic displacement in the propagating medium;
k is the wave number;
p_s' is the acoustic pressure;
t is the time;
x, y are the coordinates of the duct, axial and transverse;
v_s' is the acoustic velocity in the propagating medium;

δp_s is the pressure perturbation;

δx, $(\delta x)_0$ are the small displacement of the flame front along the x-axis, and amplitude of wave generation at the flame surface;

δv_{sx}, δv_{sy} are the velocity perturbation components;

is the wavelength at the flame surface;

ρ_s is the density;

ω is the radian frequency of acoustic vibrations.

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